Weak and Strong Sustainability, Environmental Conservation and Economic Growth

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Abstract

Weak and strong sustainability are often regarded as opposing paradigms. This is mainly a consequence of different ethical and philosophical perspectives, different axiomatic foundations of the models that are used, and different constraints that are either made explicit or implicit. This gives raise to the question about consequences of these assumptions and the question whether sustainability and economic growth foreclose each other. Correspondingly, the aim of this article is to investigate explicit and implicit assumptions in existing models of weak and strong sustainability and to examine whether the different positions that have been taken by advocates of the different paradigms can be justified on an analytical ground. For this purpose, the Solow/Hartwick model of intergenerational equity with nonrenewable resources is gradually extended to include renewable resources, endogenous technical progress, and stock pollution.

This reveals the fundamental role of endogenous technical progress for sustainable development, the inconsistency of implicit sustainability assumptions in various models, and the existence of a Hartwick rule for Daly's steady-state economy. Moreover, the analysis shows that the concepts of Solow sustainability and strong sustainability coincide as a special case of weak sustainability. The latter proves to be more comprehensive than the traditional conceptions that are based on the works by Solow and Daly, respectively. It aims at maintaining the welfare potential of an economy over time, including environmental concerns, and does not foreclose economic growth by assumption. Finally, the article addressed the consequences including issues of basic human needs and critical natural capital in our models.

Keywords: Sustainable development, natural resources, capital theory, economic growth, social welfare.

JEL classification: Q01, Q20, Q32, Q56.

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1 Introduction

Sustainable development is a global challenge which requires a progressive transformation of our economies, such as to satisfy the needs and preferences of the present generation without compromising the opportunity of future generations to meet their own needs and aspirations (WCED, 1987). This does not prescribe a fixed state of harmony, or foreclose economic growth. Rather, the idea of sustainable development leads beyond the traditional, ecologically based conception of physical sustainability to the social and economic context of development (Adams, 1990). It involves concerns for environmental preservation and economic development, and correspondingly calls for an integrated approach of evaluating trade-offs between conservation and change. This is inherently dynamic and state dependent.

Yet, differences in disciplinary perspectives, and differences in the philosophical and ethical interpretation of sustainable development have resulted in concepts of sustainability that give priority to either economic or environmental objectives, such as the opposing paradigms of "weak" and "strong" sustainability (cf. Pearce et al., 1994; Turner et al., 1994; Hediger, 1999; Neumayer, 1999). They are based on different conceptions of capital theory. On one hand, the theoretical foundation of the weak sustainability paradigm is in the neoclassical theory of economic growth and capital accumulation and its extension to include nonrenewable resources (Solow, 1974, 1986; Hartwick, 1977, 1978a). On the other hand, the paradigm of strong sustainability is grounded on the thermodynamic foundation of a steady-state economy (Daly, 1972, 1974, 1977).

Although Solow and Daly share the idea of a stationary path with constant population and constant consumption per capita, the concepts of sustainability that have been derived from their models are quite different. On one side, advocates of weak sustainability emphasize the necessity of maintaining the stock of total capital, man-made and natural—or, to use Solow's words, "an economy's generalized productive capacity" (Solow, 1986). On the other side, advocates of strong sustainability emphasize the necessity of maintaining the stock of natural capital rather than total capital as a prerequisite of sustainable development.

The distinction between the two schools of thought involves different ethical positions with respect to environmental stewardship, and divergent technological assumptions—in particular different positions regarding the question about the substitution of man-made capital for natural resources. But, there is also a common ground as regards the economic objective of constant consumption per capita and implicit assumptions in the models of Solow and Daly. Correspondingly, the aim of this article is to analyze both explicit and implicit assumptions about weak and strong sustainability in the models of Solow and Daly and extensions thereof, and to examine whether the different positions that have been taken by advocates of the different paradigms can be justified on an analytical ground. Moreover, the aim of this article is to investigate whether sustainability and economic growth foreclose each other. For this purpose, I start with a review of fundamental principles of weak and strong sustainability in Section 2. Based on this background, I present a formal analysis which includes various aspects and concerns of sustainability and development, and which allows us to investigate the role of implicit assumptions in different models of sustainability. The starting point of this analysis is the Solow/Hartwick model of capital-resource substitution and intergenerational equity (Solow, 1974; Hartwick, 1977, 1978a) which is extended in Section 3 to cover renewable resources and endogenous technical progress. In Section 4, these models are compared with a model of Daly's steady-state economy (Daly, 1972, 1977). Moreover, the problem of stock pollution is addressed in Sections 5 and 6. In the latter, the Solow/Hartwick model is further extend to the welfare context of the economy's overall capital base. The analysis is completed with considerations about basic human needs and critical levels of the ecosystem's overall integrity. Finally, Section 7 concludes.

2 Concepts of weak and strong sustainability

Although the idea of sustainability is by no means new, neither to economists nor ecologists, there is considerable disagreement about the conceptual and operational content of the term. Among other reasons, this is caused by differences in disciplinary perspectives—including different paradigms and axiomatic foundations of the dynamic models within which the concepts have been explored—, and differences in the philosophical and ethical interpretation of sustainable development (Common and Perrings, 1992). This has resulted in the above mentioned paradigms of "weak" and "strong" sustainability.

In general terms, the idea behind the paradigm of "weak" sustainability implies an economic value principle which is founded within the body of neoclassical capital theory, whereas conceptions of "strong" sustainability are based on biophysical principles. This is a result of different visions about how a sustainable world can and should look like, and how to manage change. Moreover, it is a consequence of different partial perspectives, and implies different objectives of "*what should be sustained*" and different conceptions of capital (Hediger, 1999). Apparently, this involves different minimum requirements that must be satisfied if either weak or strong sustainability objectives should be achieved.

The necessary condition for “weak” sustainability is that some suitably defined value of aggregate capital—including man-made capital and the initial endowment of natural resources—must be maintained intact over time. This can be variously defined, dependent on the objective that shall be achieved:

- In narrow terms, it can be referred to as “*very weak sustainability*”, and requires that the generalized production capacity of an economy is maintained intact, such as to enable constant consumption per capita through time (Solow, 1974, 1986). This is also referred to as “Solow sustainability” (Common and Perrings, 1992).
- In broader terms, “*weak sustainability*” requires that the welfare potential of the overall capital base remains intact (Pearce et al., 1994; Opschoor, 1996; Hediger, 1999, 2000). This is not restricted to sustaining a material standard of living, or consumption, but also includes values that are related to non-consumptive uses (existence and bequest values) and the public good character (amenity and recreational values) of the environment.

In contrast, the conception of “strong” sustainability implies a physical principle which is founded upon the laws of thermodynamics and processes of biological growth. As a basic principle of resource management, it has a long tradition in forestry, and has logically been extended to other domains of natural resource management. For instance, minimum criteria of “strong” sustainability are generally expressed in physical terms, saying that certain properties of the physical environment must be sustained. However, it is not clearly defined in the literature what it is that should be sustained.

In general, operational criteria, such as sustainable yield, can only be defined in the context of a particular problem. This ranges from simple problems of optimally harvesting one single biological resource, such as a fish population, to more complex problems of multiple populations and ecosystem management (Clark, 1990), and the maintenance of our global life-support system (Costanza et al., 1991; Costanza, 1991). The latter perspective is fundamental to ecological economics, and emerged from the basic paradigm that the economy is an open subsystem of the finite and non-growing global ecosystem, the environment (Costanza et al., 1991; Daly, 1991a, 1991b).

From a system perspective, a minimum necessary condition of “*strong sustainability*” is that the total stock of natural capital remains constant over time (Daly, 1991a). This implies an “ecological value principle” which measures the total stock of natural capital from an ecosystem perspective. This can also be referred to as a concept of “environmental quality” and represented as a function of the stocks of biological resources, ecosystem space, nutrients available, and other environmental assets that are essential for the integrity of the ecosystem, and provide use and non-use values to society (Hediger, 1998). In general, this does not imply preservation of every single asset. Nonetheless, to make strong sustainability an operational principle, several authors have translated the constant natural capital rule into a set of ecological criteria ("safe, minimum sustainability standards", SMSS) that are defined by the rate of regeneration and the assimilative capacity of the environment (Costanza, 1991; Daly, 1991a, 1991b). This is a stationary-state principle which is also referred to as "*very strong sustainability*" (Turner et al., 1994; Hediger, 1999). It does not necessarily coincide with Ciriacy-Wantrup's idea of a safe minimum standard of conservation (SMC) that is not defined as a static concept with a fixed rate of resource use. Rather, a safe minimum standard of conservation is "a given state in the intertemporal distribution of use rates" (Ciriacy-Wantrup, 1968: 54) that is realized along a socially optimal trajectory by avoiding those physical conditions (the critical zone) "which would make it uneconomical to halt and reverse depletion" (253).

In contrast to the stationary-state principle of SMSS, Ciriacy-Wantrup's concept of SMC is fundamental for sustainable development. As advocated by the WCED (1987), a minimum requirement of sustainable development is that adverse environmental impacts should be minimized such as to sustain the ecosystem's overall integrity. Apparently, this does not need that the stocks of natural resources should be preserved at their current states. Rather, it is the aggregate ecosystem structure and life-support capacity that should be taken into account (Turner et al., 1994). This also refers to the resilience of ecosystems (Common and Perrings, 1992; Perrings, 1991, 1996; Holling, 1973) and the idea of "critical natural capital" (Pearce et al., 1994), saying that the environment should not be threatened to the extent of potentially irreversible effects.

Correspondingly, sustainable development requires maintaining natural capital, or, more precisely, ecosystem capital above some critical level. I have shown elsewhere With respect to the use of the environment, this is less restrictive than the concept of strong sustainability, but more restrictive than the weak sustainability principle (Hediger, 1999). However, it is not sufficient for sustainable development since the satisfaction of basic human needs is another minimum requirement that must be met. Conventionally, these are defined in terms of adequate food and water supply, health care, shelter, and minimum education (Chichilnisky, 1977; WCED, 1987; Moon, 1991). At the aggregate level, this can be represented by some minimum income per capita which requires satisfaction of basic human needs (Atkinson et al., 1997; Hediger, 1999).

In sum, we can distinguish four key concepts of sustainability that are characterized by different minimum requirements:

- *very weak sustainability* (VWS) is characterized by constant per capita consumption;
- *weak sustainability* (WS) is characterized by some non-decreasing social welfare;
- *strong sustainability* (SS) is characterized by constant environmental quality;
- *very strong sustainability* (VSS) is characterized by a set of stationary-state conditions.

By contrast *sustainable development* (SD) requires compliance with critical levels of natural capital and basic human needs that are not addressed by notional conceptions of neither weak nor strong sustainability. Correspondingly, there is a need to investigate and compare the different models of weak and strong sustainability, and to provide insights for the further development of models that are more coherent with the idea of sustainable development.

3 Sustained-yield and Solow sustainability: models of constant consumption

The idea of sustainability as a physical principle of even flow of harvest has a long tradition in the economics and management of renewable resources. It implies a basic model of constant consumption of a natural resource which is inherent to the concept of sustainable yield in many economic models of open access fishery. However, its application is restricted to single population models, and the extension is not straightforward to the context of multi-species models (Clark, 1990). In contrast, the value principle of very weak sustainability is characterized by the maximum consumption per capita that can be maintained over time if the rents from nonrenewable resources are invested in reproducible capital (Solow, 1974, 1986). Apparently, this corresponds to Hicks' (1946) definition of income and Weitzman's (1976) conception of net national product as the largest permanently maintainable value of consumption.

Another model, which also implies this concept of constant consumption is that of Daly's (1972, 1974, 1977) steady-state economy, and which is referred to as a concept of strong sustainability (Daly, 1991a: 250). Thus, the question arises to what extent weak and strong sustainability principles are compatible with one another, and how they can be integrated in a comprehensive sustainable development framework of economy-environment interactions. Moreover, it involves the analytical question about how the principles of weak and strong sustainability are, either explicitly or implicitly, considered in environmental and resource economic models and theory.

3.1 Model I: The Solow/Hartwick model with renewable resources

The starting point of our analysis is Hartwick's (1978b) extension of the original Solow/Hartwick model with non-renewable resources (Solow, 1974; Hartwick, 1977, 1978a) to the case of renewable resources, still assuming constant population and given technology. In this case, the Hartwick rule says that, for intergenerational equity in the sense of constant consumption per capita, both the rents of non-renewable and renewable resources must be invested in reproducible capital.

The model is that of a simple, closed economy that produces one single output Y which is both used for aggregate consumption C and investment into man-made capital. The good is produced with four factors: man-made capital K, labor L which is assumed to be constant, as well as physical input flows of minerals X and harvest H from nonrenewable and renewable resources, respectively. For simplicity, we assume homogeneity of the stocks of man-made capital K, non-renewable resources N and renewable resources R, each. Furthermore, we assume that K depreciates at a given constant rate δ $(0 < \delta < 1)$. The non-renewable resource stock N declines over time due to resource extraction at the rate X, and the intertemporal change of the renewable resource stock is determined by the difference between natural regeneration $g(R)$ and the harvest rate H, where $g(R)$ is a biological growth function, as generally used in bio-economic models.[1]

The dynamics of the system are formally given by:

$$\dot{K} = Y - \delta K - C \quad , \quad \dot{N} = -X \quad , \quad \dot{R} = g(R) - H \tag{1}$$

In correspondence with the model of Hartwick (1978b), costs of extraction and harvesting are represented as shifts in the production function. A decrease in one resource stock increases the extraction or harvest cost of that resource, and, *ceteris paribus*, reduces the potential output of the economy. Therefore, the natural resource stocks N and R are included in the

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[1] The biological growth function is usually characterized as follows: $g(R) > 0$ for $0 < R < R_{MAX}$, $g(0) = 0$ and $g(R_{MAX}) = 0$, with $g'(R) > 0$ for $0 < R < R_{MSY}$, $g'(R) < 0$ for $R_{MSY} < R < R_{MAX}$ and $g''(R) < 0$ for $0 < R < R_{MAX}$, with R_{MSY} and R_{MAX} denoting the resource stocks that corresponds to the maximum sustainable yield and carrying capacity, respectively.

production function of our economy, where all inputs are assumed to be essential and substitutable for one another: $Y = f(K,L,X,H,N,R)$ with $f_v > 0$ and $f_{vv} < 0$ for $v = K,L,X,H,N,R$.

Assuming constant population and technology, the objective is to find for each time the maximum aggregate consumption that can be achieved under consideration of the dynamics of the capital and resource stocks, as described above, and of the production function $f(.)$ which is assumed to be concave and twice continuously differentiable. This optimization problem is formally represented by the subsequent current-value Hamiltonian that is to be maximized for each instant in time:

$$\Omega = C + \varphi_K\left[f(K,\bar{L},X,H,N,R) - C - \delta K\right] - \varphi_N X + \varphi_R\left[g(R) - H\right] \qquad \text{max!} \qquad (2)$$

where φ_K, φ_N and φ_R represent the shadow prices of the of the state variables K, N and R. For a given social rate of discount, $\rho > 0$, and given initial stocks of man-made capital and natural resources (K_0, N_0 and R_0), the solution of this optimization problem results in the subsequent first-order conditions for intertemporal efficiency:

$$f_K = \rho + \delta \quad , \quad \dot{f}_X = \rho f_X - f_N \quad , \quad \dot{f}_H = [\rho - g']f_H - f_R \qquad (3)$$

In addition, the Hartwick rule can be written as follows:

$$\dot{K} = f_X X - f_H\left[g(R) - H\right] \qquad (4)$$

This corresponds to the Solow sustainability requirement that the aggregate value of the total capital stock (K, N and R) must be kept constant over time. Hartwick shows that, under the given assumptions, the investment of the resource rents according to this rule theoretically entails perpetually constant consumption over time.[2]

Hartwick (1978b) provides the simplest model which integrates the theory of biological resource management with the requirements of intergenerational equity in a changing economy.[3] This does not in general involve constant stocks of renewable resources and man-

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[2] Notice that the assessment of the resource rents requires the solution of the entire infinite horizon optimization problem that is to maximize the present value of the current-value Hamiltonian Ω at every point in time.

[3] Extensions of this model to the non-autonomous case with time-dependent technology and terms of trade, and with a time-dependent discount rate, respectively, are discussed by Hartwick and Van Long (1999). The case of

made capital. Rather, dynamic efficiency requires continuous adjustment of the state and control variables along the optimal trajectories that are given in equations (1) and (3) toward the steady state $\dot{C} = \dot{K} = \dot{N} = \dot{X} = \dot{R} = \dot{H} = 0$ which, if a global optimum exists, is asymptotically achieved with time. In other words, Solow sustainability requires simultaneous optimization of the entire problem of capital accumulation, non-renewable resource extraction and biological resource management. In contrast, more simplified models that explicitly or implicitly assume constant stocks of renewable resources or of man-made capital do not in general satisfy the requirements of intertemporal efficiency. The latter describes a time path which avoids wasteful uses of scarce resources and, therefore, constitutes a prerequisite for sustainable development.

A comparison with the original Solow/Hartwick model

As pointed out by Hartwick (1978b), a modification of the above model would imply the artificial assumption of a constant stock of renewable resources, and thus the sustainable harvest rule $H = g(R)$. Abstracting form other environmental assets, this corresponds to the *strong sustainability* requirement of maintaining the stock of ecological capital constant over time. In other words, the original model of Solow (1974) and Hartwick (1977) is characterized by an implicit strong sustainability criterion and by the original Hartwick rule which requires investment of the rents from non-renewable resources in man-made capital:

$$\dot{R} = 0 \quad \textit{and} \quad \dot{K} = f_X X \tag{5}$$

Apparently, this does not entirely satisfy the conditions of intertemporal efficiency. Thus, sustainable resource management and the investment of resource rents according to equation (5) are not in general sufficient for sustainable development. Rather, it constitutes a second-best solution, since the ecological sustainability criterion implies a reduction of the social opportunity space.

Likewise, the popular idea of simply substituting renewable resources for non-renewable ones with constant consumption and a constant aggregate stock of capital implies a reduction of the social opportunity space for sustainable development, in comparison with the unconstrained case which is represented in equation (2).

economic sustainability with renewable resources and heterogeneous time preferences is addressed in Li and Löfgren (2000).

3.2 Model II: The Solow/Hartwick model with modified technological assumptions

The concept of Solow sustainability is subject to criticism because of the assumed substitutability of man-made capital for natural resources (e.g., Daly, 1991a; Victor, 1991). In general, this criticism on the Solow/Hartwick-type model refers to the Cobb-Douglas representation of an economy's production function which implies that natural resources can asymptotically be replaced by man-made capital, and that therefore the potential for substitution never diminishes. However, as pointed out by Pearce et al. (1994), the crucial question is not the form of the production function per se, but rather the feasibility to replace manufactured capital for natural capital. This potential of capital substitution will be limited because natural resources are required to manufacture capital and consumer goods. Therefore, the success of any attempt to substitute man-made capital for natural resources will be limited by the extent to which an increase in manufactured capital requires an input of natural resources (Victor, 1991). However, this does not provide an argument against the use of neoclassical production functions and capital theory. Rather, the context must be taken into consideration.

Limited substitutability of production factors

Daly (1991a: 250), for instance, argued that it is not sufficient to protect the overall value of man-made and natural capital, rather natural capital must be protected separately, because man-made capital and natural resources are not substitutes but complements in most production functions. His argument is that man-made capital and labor are required to transform raw materials into final goods and services, and that neither material nor energy can be created nor destroyed (first law of thermodynamics). Correspondingly, there is a complementary relationship in production between the industry of man-made capital and labor, on one side, and natural resources, on the other side. Formally, this can be represented with a limitational production function, which, like the Cobb-Douglas function, is a special case of the more general family of constant elasticity of substitution (CES) production functions. The limitational form is characterized by an elasticity of substitution which is equal to zero, and may be adequate to represent technological relationships for certain types of machines or production processes. However, this will not in general be adequate for analyses at an aggregate level where possibilities of process substitution exist. An aggregate production function will therefore consist of a combination of limitational processes. If a sufficiently large number of alternative processes exists, this can be approximated by a neoclassical production function which implies some minimum factor input requirements.

From this perspective, CES production functions with an elasticity of substitution between 1 and 0 are adequate to integrate the issues of process substitution and minimum factor requirements in the extreme case of substitution. This has, for instance, been used by Manne (1979) in developing *ETA Macro*, a techno-economic energy model for the United States. He considered two pairs of input in his economy-wide production function: capital and labor, on one hand, and non-electric and electric energy, on the other hand. Moreover, Manne assumed constant returns to scale in all four inputs, unit elasticity of substitution between capital and labor as well as between non-electric and electric energy, and a constant elasticity of substitution $\sigma = 0{,}25$ between the two pairs of inputs.

In a similar way, our aggregate production function with four inputs can be represented as a CES function with an elasticity of substitution $\sigma\,(0 < \sigma < 1)$ and scale elasticity $\varepsilon\,(0 < \varepsilon \leq 1)$:[4]

$$Y = \left[a \cdot \left(K^{\alpha} \cdot L^{1-\alpha}\right)^{\frac{\sigma-1}{\sigma}} + b \cdot \left(X^{\beta} \cdot H^{1-\beta}\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\varepsilon\sigma}{\sigma-1}} \tag{6}$$

In this form, total output Y is an increasing function of man-made capital, labor, and flows of non-renewable and renewable resources. The two pairs of inputs—capital and labor, on the one side, and non-renewable and renewable inputs, on the other side—are each considered as close substitutes, while the elasticity of substitution between the two pairs may be relatively low, as in Manne's *ETA Macro* model.

This production function can be translated into a more general form and extended to include costs of resource extraction and harvesting, and the state of technology J which is considered as an additional factor that increases the economy's output potential:

$$\begin{gathered} Y = Y(f(K,L), M(X,H), N, R, J) \\ with \quad Y_f\,,\, Y_M\,,\, Y_N\,,\, Y_R\,,\, Y_J\,,\, f_K\,,\, f_L\,,\, M_X\,,\, M_H > 0 \\ Y_{ff}\,,\, Y_{MM}\,,\, Y_{NN}\,,\, Y_{RR}\,,\, Y_{JJ}\,,\, f_{KK}\,,\, f_{LL}\,,\, M_{XX}\,,\, M_{HH} < 0 \end{gathered} \tag{7}$$

[4] A slightly different formulation is provided by Smulders and de Nooij (2003) with the concept of *effective labor* and *effective energy* inputs, as an expression of the fact that the gross complements of labor and energy inputs are each combined with complementary intermediate inputs ("capital"). This is included in an aggregate CES production function, where the level of technology is expressed by the average quality of labor-related and energy-related inputs, respectively.

This production function allows for gradual substitution between the different inputs, but not for perfect substitution of capital and labor for natural resources. In addition, it explicitly includes the state of technology as a factor of production, which must not be constant but can change over time.

Endogenous technical progress

In the original Solow/Hartwick model, technology is autonomous and assumed to be constant. Solow (1974) argued that, in the case of unlimited technical progress and constant population, the Rawlsian minimax criterion of intergenerational equity may be unsatisfactory, because "it requires society to choose a constant level of consumption per head when it could have exponentially-growing consumption per head." Yet, this may alter if technical progress is endogenous.

In a simple form, endogenous technical progress can be represented as:

$$\dot{J} = \alpha I - \beta J \qquad , \; 0 < \alpha < 1 \, , \, 0 < \beta < 1 \tag{8}$$

where α is the research success coefficient, and β the rate of decay in technical knowledge.

The accumulation of knowledge is not without cost. It depends on the amount of resources devoted to research and education, and thus the sacrifice of final output that would be required to enable technological progress. This is represented in our model by the rate of investment I in inventive activities. Correspondingly, the net accumulation of manufactured capital is:

$$\dot{K} = Y - C - I - \delta K \tag{9}$$

The overall problem remains in principle the same as in the first model. It is to find the maximum feasible rate of consumption. This is restricted by the dynamic constraints that are given in (1), (8) and (9) for the state variables N, R, J and K, as well as the production function (7).

Assuming constant population and labor input, the optimization problem for each time is formally represented by the extended current-value Hamiltonian:

$$\begin{aligned} \Omega^{+} = C + \varphi_K \left[Y(f(K,\overline{L}), M(X,H), N, R, J) - C - I - \delta K \right] \\ - \varphi_N X + \varphi_R \left[g(R) - H \right] + \varphi_J \left[\alpha I - \beta J \right] \qquad \text{max!} \end{aligned} \tag{10}$$

with φ_K, φ_N, φ_R, φ_J denoting the shadow prices of the state variables K, N, R and J.

Using the first order optimality conditions and the subsequent set of definitions $Y_K \equiv Y_f f_K$, $Y_X \equiv Y_M M_X$ and $Y_H \equiv Y_M M_H$, the extended Hartwick rule with stocks of man-made capital, non-renewable and renewable resources, as well as endogenous technological progress can be written as:

$$\dot{K} + \frac{1}{\alpha}\dot{J} = Y_X X + Y_H \left[H - g(R)\right] \tag{11}$$

This equality corresponds to the requirement of Solow sustainability, saying that the aggregate value of the total capital stock (the economy's generalized production capacity) must be maintained intact over time, such as to enable a constant flow of consumption per capita. It requires that all resource rents must be invested in reproducible capital; this is, into manufactured capital and the state of technology, K and J.

Thus, it can be shown that, even if we introduce endogenous technological progress in the original Solow/Hartwick model, investing resource rents according to the extended Hartwick rule (11) results in a constant flow of consumption over time. This is the maximum of consumption that can be achieved at any time, given the dynamic conditions included in the optimization problem (10). Moreover, the analysis illustrates the potential role of investing in technical progress, as an efficient measure for achieving sustainable development.

4 Daly's steady-state economy and Solow sustainability: a comparison

The role of technical progress, or knowledge accumulation, has already been emphasized by Daly (1972, 1977) in his work on the steady-state economy. He defined it as a physical concept which is characterized by constant stocks of people and artifacts (wealth), maintained at some desirable and sufficient level by a minimum rate of throughput. For Daly, the latter is the maintenance cost of the stock, expressed in physical units. It is a flow which begins with depletion of resources and ends with an equal amount of waste or pollution. Correspondingly, Daly concludes that stocks and throughput are limited by space, and by environmental and resource capacities of the earth ("by the mass of the earth, by heat release, and far more important by the intricate web of ecological relationships"). Within these physical limits, however, Daly (1972) emphasized that "the want-satisfying capacity may forever increase due to increasing knowledge and technical progress."

In contrast, Solow (1974) stressed that ongoing technical progress would be unfair according to the Rawlsian criterion of intergenerational equity, since it would favor the future over the present. Yet, in his interpretation Solow (1986) is not fixed on constant technology. Rather, he emphasizes that if one generation owes anything to its successors it owes generalized productive capacity or access to a certain standard of living or a certain level of consumption. In this context, Solow (1986: 142) points out that “Whether productive capacity should be transmitted across generations in the form of mineral deposits or capital equipment or technological knowledge is more a matter of efficiency than of equity.”

Thus, the question arises, to what extent the models of Solow (1974, 1986) and Daly (1972, 1974) imply real differences with respect to sustainable development, and on whether a Hartwick-type investment rule can be applied to Daly’s steady-state economy. To this end, we use the above model II and distinguish three special cases with different assumptions about the state variables in this reference model.

Case 1**:** Constancy of population, technology and the stock of renewable resources:

$$\dot{K} = Y_X X \qquad \textit{with} \quad \dot{R} = \dot{J} = 0 \tag{12}$$

This corresponds to the original Hartwick rule and formulation of the original Solow/Hartwick model (Solow, 1974; Hartwick, 1977). It requires investment of rents from exhausting non-renewable resources in the net accumulation of manufactured capital. The unspoken assumption of a constant stock of renewable resources implies, as discussed in Section 3.1, a strong sustainability rule of even-flow harvest: $H = g(R)$. The requirement of constant technology in the original model of Solow (1974) refers to the case with autonomous technical progress. However, this assumption could be relaxed if endogenous technical progress is taken into consideration. In this case, constant technology requires, according to our formulation in equation (8), a constant flow of investment $I = (\beta/\alpha) \cdot J$, which has not been considered in the original Solow/Hartwick model.

Case 2**:** Constancy of population, manufactured capital and the stock of renewable resources:

$$\dot{J} = \alpha Y_X X \qquad \textit{with} \quad \dot{R} = \dot{K} = 0 \tag{13}$$

This is a modification of the Hartwick rule which requires exclusive investment of rents from exhausting non-renewable resources in technological progress. In this case, human capital and technical knowledge are substitutes for non-renewable resources. These assumptions correspond to the requirements of Daly’s steady-state economy, which is defined as an economy with constant stocks of people and artifacts. Yet, the maintenance of the stock of manufactured capital also requires investments to replace depreciated capital δK.

Case 3**:** One can also translate Solow's (1986) gradual extension with respect to technology into a modified Hartwick rule, which requires investment of the rents from non-renewable resources into any form of reproducible capital, this is into manufactured capital plus technological knowledge:

$$\dot{K} + \frac{1}{\alpha}\dot{J} = Y_X X \qquad\qquad with \quad \dot{R} = 0 \tag{14}$$

This illustrates that even the original rule of Solow and Hartwick might be interpreted in a more flexible way if endogenous technical progress is taken into consideration as a part of the economy's total productive capacity (stock of economic capital).

Table 1: Explicit and implicit sustainability assumptions on harvesting and investment

Model / case:		Con-sumption	Renewable resources	Manufactured capital	Non-renewable resources	State of technology
—	Sustained yield	$\dot{C} = 0$	$H = g(R)$	—	—	—
Case 1	Solow (1974)	$\dot{C} = 0$	$H = g(R)$	$\dot{K} = Y_X X$		$I = (\beta/\alpha)J$
Case 2	Solow (1986)	$\dot{C} = 0$	$H = g(R)$	$\dot{K} + \alpha^{-1}\dot{J} = Y_X X$		
Case 3	Daly (1972, 1977)	$\dot{C} = 0$	$H = g(R)$	$\dot{K} = 0$	$\alpha Y_X X = \dot{J}$	
Model I	Hartwick (1978b)	$\dot{C} = 0$	$\dot{K} = Y_X X + Y_H[H - g(R)]$			$I = (\beta/\alpha)J$
Model II	Equation (11)	$\dot{C} = 0$	$\dot{K} + \alpha^{-1}\dot{J} = Y_X X + Y_H[H - g(R)]$			

A comparison of these three cases with the models I and II from Section 3 is given in Table 1. It shows that Daly's steady-state economy (SSE) is a special case of Solow's (1986) interpretation of intergenerational equity with modified technological assumptions (case 3). Yet, despite Daly's emphasis on a constant stock of artifacts, there is a need of investment in manufactured capital in his steady-state economy such as to maintain the stock and replace depreciated capital δK.

Driven by technological progress, the stock of manufactured capital cannot be homogenous. Rather, with the realization of technical progress, the economy may invest in the most recent technology vintages and continuously replace the oldest ones. This finding is not restricted to the case of Daly's steady-state economy. It is also crucial for the general case with endogenous progress, and therefore for more realistic models of sustainable development.

Moreover, the analysis shows that both Solow's conceptions of intergenerational equity and Daly's steady-state economy are restricted cases of *very weak sustainability*. Both assume constant population and imply a constant stock of renewable resources. In both cases, the objective is to maintain a constant level of consumption per capita over time. The justifications for zero growth, however, are different. For Solow, it follows from the ethical requirement of intergenerational equity (Rawlsian justice across generations). For Daly, it is a consequence of the physical limits of the global environment system. As an advocate of strong sustainability, he requests that both man-made and natural capital must be maintained intact separately.

Despite these differences in paradigms, no clear conclusion can be drawn on whether the consumption levels are the same or not. From an economic point of view, the optimal level of consumption is determined by the current level of Hicksian income, this is the maximum level of consumption that can be maintained over time (Hicks, 1946). This is sustainable by definition (Daly, 1991a) and requires an initial capital stock big enough to support a decent standard of living, else it would perpetuate poverty—but this rule "can not tell us why the initial capital stock should ever have been accumulated" (Solow, 1974).

In contrast, Daly's steady-state economy is defined as "an economy with constant stocks of people and artifacts, maintained at some desired, sufficient levels" (Daly, 1991a). This can also be translated into the objective of a constant level of consumption per capita, which, according to Daly, is to be maintained by a minimum flow of throughput. This, however, does not make the difference to neoclassical models of economic growth. The original Solow/Hartwick model can also be formulated such as to minimize the use of non-renewable resources, given a fixed level of consumption (Sato and Kim, 2002).

Thus, the main differences between the contributions of Solow and Daly are a result of the divergent ethical and philosophical foundations of the constant consumption path and the different treatment of manufactured capital and technology. As illustrated in Table 1, both can be characterized as special cases of a more general model that is presented in Section 3. Both require the investment of resource rents into some suitably defined substitutes, and both can be formally analyzed in a throughput minimizing framework. Moreover, as a matter of simplification, both imply very restrictive assumptions with respect to the environment. Both neglect renewable resource harvesting and pollution, or imply by construction of their models constant stocks of renewable resources and pollution. The latter issue is examined in the following section.

5 The Solow/Hartwick model with stock pollution

The extension of the models of Solow, Hartwick and Daly to include stock pollutants is straightforward. In order to compare with the results from the original Solow/Hartwick model, we start again with the model of Hartwick (1978b), but assume that the aggregate output Y is affected by the stock of pollution S:

$$Y = f(K,\overline{L},X,H,N,R) - D(S) \tag{15}$$

$D(S)$ is the physical damage function which is progressively increasing with the stock of pollution: $D' > 0$, $D'' > 0$. This problem arises due to pollutant accumulation:

$$\dot{S} = E(X,H) - V(Z) - A(S) \tag{16}$$

which is the net effect of emissions $E(X,H)$, pollution abatement $V(Z)$, and natural self-purification within the ecosystem's assimilative capacity $A(S)$. Thus, three additional functions need to be specified for the analysis of the pollution control problem.

First, emissions result as by-product of transforming natural resources, X and H, in the production process. Due the thermodynamic law of "conservation of mass" and the usually assumed decreasing marginal productivity of material input, the emission function $E(X,H)$ is convex: $E_X, E_H > 0$, $E_{XX}, E_{HH} > 0$, $E_{XX}E_{HH} - (E_{XH})^2 \geq 0$.

Second, the self-purification capability of the environment, the assimilative capacity $A(S)$, is crucial for optimal pollution control. It can have different functional forms, such as discussed by Elliott and Yarrow (1977), Hediger (1991), Cesar and de Zeeuw (1995), Pezzey (1996), and Toman and Withagen (2000). For the general purpose of this paper, we do not need to fully specify the functional form of this essential process. It is sufficient to notify the impact of the assimilation process upon the dynamics of pollutant accumulation, as given in equation (16), and that the function is concave: $A''(S) \leq 0$.

Finally, pollution abatement is a concave function ($V' > 0$, $V'' < 0$) of factor inputs Z that are no longer available for consumption and investment. Correspondingly, the expression for capital accumulation from equation (1) must be reformulated as follows:

$$\dot{K} = Y - \delta K - C - Z \tag{17}$$

The objective of the intertemporal allocation problem remains in principle the same as in the original case without pollution. It is to find for each time the maximum aggregate level of consumption, assuming constant population and technology. It requires the solution of the modified optimal control problem which is formally represented by the current-value Hamiltonian:

$$\begin{aligned} \Omega^{++} &= C + \varphi_K\left[f(K,\overline{L},X,H,N,R) - D(S) - C - Z - \delta K\right] \\ &\quad - \varphi_N X + \varphi_R\left[g(R) - H\right] + \varphi_S\left[E(X,H) - V(Z) - A(S)\right] \quad \text{max!} \end{aligned} \tag{18}$$

As in the original model (2), the costate variables φ_K, φ_N and φ_R represent the shadow prices of K, N and R. However, their formal representation partly changes in comparison with the conditions in equation (3). In the presence of stock pollution, the first-order conditions are:

$$\begin{gathered} \varphi_K = 1 \ , \quad \varphi_N = f_X + \varphi_S E_X \ , \quad \varphi_R = f_H + \varphi_S E_H \\ f_K = \rho + \delta \ , \quad \dot{\varphi}_N = \rho\varphi_N - f_N \ , \quad \dot{\varphi}_R = \varphi_R\left[\rho - g'\right] - f_R \\ \dot{\varphi}_S = \varphi_S\left[\rho + A'\right] + D' \ , \quad \varphi_S = -(1/V') < 0 \end{gathered} \tag{19}$$

with φ_S denoting the shadow price of pollution, which is negative since pollution is a "bad".

Thus, to achieve an optimal allocation the external costs of pollution must be internalized. The resulting shadow prices φ_N and φ_R of the natural resources and related resource inputs in production are lower than without pollution. Moreover, under consideration of stock pollution, the Hartwick rule from equation (4) must be extended:

$$\dot{K} = \varphi_N X - \varphi_R \dot{R} - \varphi_S \dot{S} = f_X X - f_H \dot{R} + \frac{1}{V'}[\dot{S} - E_X X + E_H \dot{R}] \tag{20}$$

It includes an additional term that does not vanish even if the stock of pollution would be kept constant over time, which is implicit in the original models of Solow, Hartwick and Daly. Thus, equation (20) elucidates that stock pollution cannot be excluded from our models by assumption. Rather, to comprehensively address the challenge of sustainable development and to be inherently consistent the models of Solow, Hartwick and Daly must be extended. They must explicitly address the accumulation and decay of pollutants, even in a VWS framework which does not consider environmental benefits from resource stocks and direct disutility from pollution.

6 Maintaining the social welfare potential

So far we have been restricted to models of very weak sustainability with constant consumption and a reduction of the environment to functional values in production. In contrast, the concept of *weak sustainability* is more comprehensive. It is an integrative principle which requires that the total value of aggregate economic activity and environmental quality should be maintained intact over time (Hediger, 2000). This does not need that either stock of natural capital or man-made capital should be kept constant over time. Rather, the rationale is that some suitably defined value of services of these stocks should be sustained, and that changes in environmental quality can be traded-off against changes in income, and vice versa. By this means, we can take into account that, apart from instrumental values in the economy's production function, the environment provides non-consumptive services to present and future generations, such as recreation and amenity services, or existence and bequest values. In other words, the concept of weak sustainability integrates environmental benefits (value of the stock of renewable resources and disutility from pollution) with the benefits of economic development (value of the economy's production capacity). This can formally be expressed in a social welfare function.

The optimal allocation problem

Assuming constant population, we use in the subsequent analysis a *social welfare* measure U that is represented by a strictly concave and twice continuously differentiable function of the current values of aggregate consumption C and environmental quality Q:

$$\begin{gathered} U = U(C,Q) \\ U_C, U_Q > 0\,,\; U_{CC}, U_{QQ} < 0\,,\; U_{CC}U_{QQ} - (U_{CQ})^2 \geq 0\,, \\ \lim_{C\to 0} U_C(C,Q) = \infty\,,\; \lim_{Q\to 0} U_Q(C,Q) = \infty \end{gathered} \tag{21}$$

In addition, *environmental quality* Q is represented as a strictly concave and twice continuously differentiable function of the current stocks of renewable resources and pollution, R and S.

$$\begin{gathered} Q = Q(R,S) \\ Q_R > 0\,,\; Q_{RR} < 0\,;\; Q_S < 0\,,\; Q_{SS} < 0\,;\; Q_{RR}Q_{SS} - (Q_{RS})^2 \geq 0 \end{gathered} \tag{22}$$

Now, the question is whether, in this more comprehensive framework, environmental conservation (strong sustainability) and economic growth (very weak sustainability) can be compatible with each other, and whether the investment of resource rents can bring about a constant or even increasing flow of welfare (weak sustainability). To this end, we maximize over an infinite time horizon the present value of the above utility function subject to the same set of constraints that has been defined in the previous sections:

$$\begin{gathered}\dot{K} = Y - C - I - Z - \delta K \quad , \quad \dot{J} = \alpha I - \beta J \\ \dot{N} = -X \quad , \quad \dot{R} = g(R) - H \quad , \quad \dot{S} = E(X,H) - V(Z) - A(S) \\ Y = Y(f(K,L), M(X,H), N, R, J) - D(S)\end{gathered} \tag{23}$$

This intertemporal allocation problem can formally be represented by the subsequent current-value Hamiltonian that is to be maximized at each time, with φ_K, φ_N, φ_R, φ_J and φ_S again denoting the shadow prices of the state variables K, N, R, J and S:

$$\begin{aligned}\tilde{\Omega} = U\big(C, Q(R,S)\big) + \varphi_K\big[Y(f(K,\bar{L}), M(X,H), N, R, J) - D(S) - C - I - Z - \delta K\big] \\ - \varphi_N X + \varphi_R\big[g(R) - H\big] + \varphi_J\big[\alpha I - \beta J\big] + \varphi_S\big[E(X,H) - V(Z) - A(S)\big] \qquad \max!\end{aligned} \tag{24}$$

Intertemporal efficiency requires that the first order optimality conditions are satisfied, with ψ denoting society's utility discount rate ($\psi > 0$):

$$\begin{gathered}\varphi_K = U_C = \alpha\varphi_J = -\varphi_S V' \quad , \quad \varphi_N = \varphi_K Y_M M_X + \varphi_S E_X \quad , \quad \varphi_R = \varphi_K Y_M M_H + \varphi_S E_H \\ \dot{\varphi}_K = \varphi_K\big[\psi + \delta - Y_f f_K\big] \quad , \quad \dot{\varphi}_J = \varphi_J\big[\psi + \beta\big] - \varphi_K Y_J \\ \dot{\varphi}_N = \varphi_N \psi - \varphi_K Y_N \quad , \quad \dot{\varphi}_R = \varphi_R\big[\psi - g'\big] - U_C Y_R - U_Q Q_R \\ \dot{\varphi}_S = \varphi_S\big[\psi + A'\big] + \varphi_K D' - U_Q Q_S\end{gathered} \tag{25}$$

These conditions reveal that—in contrast to the previous models of very weak sustainability—the shadow price of capital, φ_K, is no longer unit, but must be equal to the marginal utility of consumption, U_C. Moreover, due to the consideration of direct and indirect environmental benefits, the shadow prices of the renewable resource and pollution are higher than in the previous models of very weak sustainability. This theoretically results in more conservative uses of natural resources and of the environment as receptor of pollutants. Correspondingly, environmental concerns are better addressed in a model of welfare maximization which implies an economic value principle, than in models of solely maximizing the largest permanently maintainable level of consumption that foreclose economic growth for ethical reasons (Solow, 1974; Daly, 1972, 1974).

Yet, in the present model, neither environmental conservation nor economic growth must be excluded a priori. Rather, the objective of welfare maximization allows for a trade-off between economic and environmental concerns. This raises the question whether economic growth must go along with environmental degradation, or not.

Optimal capital accumulation, economic growth and strong sustainability

Economic growth is driven by the dynamic process of capital accumulation which must satisfy a set of intertemporal efficiency requirements that are expressed in terms of shadow prices. As presented in (25), the shadow price of manufactured capital, φ_K, must evolve along an intertemporal trajectory which satisfies the subsequent conditions (with $Y_K \equiv Y_f f_K$):

$$\frac{\dot{\varphi}_K}{\varphi_K} = \frac{U_{CC}\dot{C} + U_{CQ}\dot{Q}}{U_C} = \psi + \delta - Y_K \tag{26}$$

and

$$\frac{\dot{\varphi}_K}{\varphi_K} = \frac{\dot{\varphi}_J}{\varphi_J} = \psi + \beta - \alpha Y_J \tag{27}$$

Apparently, φ_K is constant only if the net social marginal productivity of both manufactured capital and technology (knowledge) are equal to the social utility discount rate ψ:

$$\dot{\varphi}_K = \dot{\varphi}_J = 0 \qquad \Leftrightarrow \qquad Y_K - \delta = \alpha Y_J - \beta = \psi \tag{28}$$

Moreover, equations (26) and (27) indicate that the *Ramsey rule* of optimal consumption growth must be extended to account for the change in environmental quality. Hence, constant consumption can only be intertemporally efficient under more restrictive conditions than the conventional Ramsey rule says. It requires the marginal productivity of manufactured capital and knowledge as well as environmental change be jointly taken into consideration:

$$\dot{C} = \frac{U_C \cdot (\psi + \delta - Y_K)}{U_{CC}} - U_{CQ}\dot{Q} \begin{cases} >0 \\ =0 \\ <0 \end{cases} \quad if \quad \frac{U_C \cdot (\psi + \delta - Y_K)}{U_{CC}} \begin{matrix} > \\ = \\ < \end{matrix} U_{CQ}\dot{Q} \tag{29}$$

and

$$\dot{C} = \frac{U_C \cdot (\psi + \beta - \alpha Y_J)}{U_{CC}} - U_{CQ}\dot{Q} \begin{cases} >0 \\ =0 \\ <0 \end{cases} \quad if \quad \frac{U_C \cdot (\psi + \beta - \alpha Y_J)}{U_{CC}} \begin{matrix} > \\ = \\ < \end{matrix} U_{CQ}\dot{Q} \tag{30}$$

This show that economic growth is intertemporally optimal and can be realized, as long as certain conditions fulfilled with respect to the marginal utility of consumption, the marginal productivity of manufactured and knowledge capital, environmental change and the current state of consumption and the environment. If under these conditions not only consumption but also social welfare $U(C,Q)$ increases, then the development path of the economy can be said to be weakly sustainable without investing resource rents into reproducible capital. The latter is indicated only if social welfare would decline along the intertemporally optimal trajectory.

Yet, advocates of strong sustainability, such as Daly (1991b), criticize this result and request that, apart of total capital (and thus social welfare), also natural capital must be maintained intact. Formally, this can be expressed with the subsequent *strong sustainability* constraint which requires that the total stock of ecological or ecosystem capital Q does not decline over time:

$$\dot{Q} = Q_R \dot{R} + Q_S \dot{S} = 0 \tag{31}$$

In this case, equations (29) and (30) can be simplified and the usual Ramsey rule applies. Thus, given sufficient substitution possibilities, strong sustainability can go along with economic growth. However, it does not necessarily imply economic growth or at least a constant flow of consumption per capita. Rather, given the above assumptions and requirements for intertemporal efficiency, the VWS condition of non-declining consumption per capita is satisfied only if the net social marginal productivity of capital is equal to or larger than the social utility discount rate ψ. Formally this is expressed by:[5]

$$\dot{C}\Big|_{\dot{Q}=0} = \frac{U_C}{U_{CC}}\left[\psi + \delta - Y_K\right] \geq 0 \qquad \text{if} \quad Y_K - \delta \geq \psi \tag{32}$$

This reveals that, given our production function with multiple inputs and the trade-off between consumption and environmental quality in the objective function, a constant flow of per-capita consumption cannot in general be achieved with a constant stock of manufactured capital, since a constant value of Y_K does not follow from $\dot{K} = 0$. The same is true with respect to endogenous technical progress. Y_J does not necessarily remain constant if $\dot{J} = 0$. Thus, neither Daly's steady-state economy nor Solow's specification of a stationary path

[5] Notice that $U_C > 0$ and $U_{CC} < 0$.

constitute adequate models of sustainable development, if the environment has a finite value; this is, if a trade-off exists in the social value function between environmental benefits and income.

Nonetheless, economic growth and strong sustainability can be compatible with each other. Under the assumption of decreasing marginal utility with consumption ($U_C > 0$, $U_{CC} < 0$), economic growth requires sufficiently large values of the marginal productivity of manufactured capital and technology:

$$Y_K \geq \psi + \delta \qquad \text{and} \qquad Y_J \geq (\psi + \beta)/\alpha \tag{33}$$

This is the case if the stocks of manufactured capital and knowledge are below their long-term optima, and if correspondingly some further accumulation of manufactured capital and knowledge is required for intertemporal efficiency. Thus, strong sustainability does not necessarily foreclose economic growth, and vice versa. However, the consistency of very weak and strong sustainability requires initial conditions that call for further investment and a social welfare function that implies a trade-off between environmental quality and income. Otherwise, economic growth would go along with degradation of the ecological capital, and environmental conservation with a decline of consumption.

Weak sustainability and the investment of resource rents

The above considerations are only based on the Ramsey rule for optimal economic growth, but do not include ethical aspects of intertemporal equity, such as the maintenance of the social welfare potential over time. Apparently, the latter requires that the aggregate value of the total capital stock must be maintained over time. This implies the extended *Hartwick rule*:

$$\varphi_K \dot{K} - \varphi_N X + \varphi_R \dot{R} + \varphi_S \dot{S} + \varphi_J \dot{J} = 0 \tag{32}$$

which in principle remains the same as in the previous sections. It requires the investment of resource rents into manufactured capital and technological progress, which, assuming constant population, results in a constant stream of social welfare over time. Under consideration of the first-order optimality conditions, the latter corresponds to the largest permanently maintainable level of social welfare.

Furthermore, it follows from the condition for constant welfare

$$\dot{U} = U_C \dot{C} + U_Q \dot{Q} = 0 \tag{33}$$

that, in a weak sustainability framework with given population, constant ecological capital Q implies constant consumption, and vice versa. Thus, the economic principle of VWS and the ecological principle of SS coincide as a special case of WS, and the concepts of Solow sustainability and Daly's steady-state economy correspond to the stationary case of the WS concept. In general, this is only asymptotically achieved with time. Thus, the conditions of Solow sustainability and Daly's steady-state economy are not as a rule compatible with the conditions of intertemporal efficiency, and therefore not sufficient for sustainable development. Rather, the use of a welfare maximizing framework is indicated as the most appropriate reference for analyzing optimal resource allocations toward sustainable development. However, our models must be further extended to include fundamental aspects of sustainable development, such as the satisfaction of basic needs and the maintenance of the ecosystem's overall integrity.

Critical natural capital and basic human needs

A minimum requirement of sustainable development is to meet a sufficient level of consumption; this is, to satisfy at least basic human needs (WCED, 1987). It implies minimum standards that cannot be traded off against each other and against market commodities without threatening survival (Pearce and Turner, 1990). In a highly aggregate form, as in our model, this can be represented by a minimum level of per-capita consumption which should not be curtailed at any time.

Another requirement of sustainable development is to "sustain the ecosystem's overall integrity" (WCED, 1987: 46). This does not necessarily imply conservation of the environment at the current state. Rather, it implies sustainability constraints that will restrict, to some degree, resource using economic activities within bounds thought to be consistent with ecosystem stability and resilience (Common and Perrings, 1992; Turner et al., 1994; Perrings, 1996), and refers to some concept of "critical natural capital" (Pearce et al., 1994). In general, this requires to maintain the overall quality of the environment (ecosystem capital) above a critical level that would still enable the system to adapt to changing conditions (resilience, integrity of the ecosystem), rather than to preserve the system in a steady state. In our simple model where the ecosystem is reduced to one single renewable resource stock, an ecological minimum requirement of sustainable development can be introduced in terms of a critical level of this stock that must be respected at any time. Conventionally, one may think of introducing such limits as additional constraints in an intertemporal allocation model.

However, the crux with such an approach is that the shadow price of each constraint remains zero unless the constraint is binding; this is, unless the critical limit is achieved.[6]

Further issues that are also relevant in this context of basic needs and critical limits of natural capital, but have not yet gained much attention in formal analyses of sustainable development and in the conceptualization of sustainability terms are population growth and land use change. Reference models are those of Pender (1998) and Krutilla and Reuveny (2002) who consider population growth in their model, as well as Hartwick et al. (2001) and Barbier (2000) who address deforestation and development in a small open economy and developing countries, respectively. Together with the inclusion of stock pollution at both the local and global scale (e.g. local water pollution and global warming), these issues must be further analyzed within a weak sustainability framework that suitably includes critical limits of ecosystems and basic human needs.

7 Conclusion

Weak and strong sustainability are considered in the literature as opposing paradigms. This is a consequence of different ethical and philosophical perspectives and different axiomatic foundations of the models upon which these concepts of sustainability are founded. On one side, very strong sustainability (VSS) calls for the strict preservation of every single environmental asset. On the other side, very weak sustainability (VWS) requires a constant level of consumption per capita over time. In between, the more moderate concepts of strong sustainability (SS) and weak sustainability (WS) that either require that the overall integrity of the ecosystem (the stock of ecological capital—SS) or the social welfare potential (the total stock of man-made and natural capital—WS) is maintained over time.

The analysis of different models of sustainability shows that VWS and SS must not necessarily be in conflict with each other. Rather, the preservation of environmental quality and economic growth can be compatible in a welfare maximizing framework, as long as further capital accumulation and technical progress is required along the intertemporally efficient trajectory. Moreover, VWS and SS coincide as a special case, a stationary case, of

[6] In principle, this analytical problem could be resolved with the use of a social value function that anticipates potentially irreversible changes that may appear beyond these critical limits (Hediger, 1999, 2000), or by using a suitably adapted goal programming approach such as used by Duraiappah (1993) to analyze issues of global warming and economic development.

WS since the latter allows for a trade-off between economic growth and environmental protection. In other words, both Solow's (1974) conception of intergenerational equity and Daly's (1972, 1977) steady-state economy constitute a stationary-state path within a WS model, given constant population.

In this article, the original Solow/Hartwick model has been gradually extended to include renewable resources, endogenous technical progress, and stock pollution. This enables an analysis which elucidates the role of endogenous technical progress and the investment in man-made capital as important elements for sustainable development, both in a VWS model with constant consumption and in a more comprehensive model of WS where economic growth is not foreclosed by assumption. Moreover, these extensions of the original Solow/Hartwick model allow us to compare Daly's model of a steady-state economy with different forms of the Solow/Hartwick model of intergenerational equity. First, it proves the existence of a Hartwick-type investment rule for Daly's steady-state economy. Second, it shows that, due to their restrictive assumptions, the models of Solow, Hartwick and Daly are special cases of a more comprehensive model of sustainable development.

Moreover, the analysis shows that the discussion of sustainability models cannot be restricted to the explicit differences that are usually pointed out by their authors and commentators. Rather, implicit assumptions must be made explicit in order to examine the implications and relevance of simplifying assumptions, such as those of constant technology in the Solow/Hartwick model or a constant stock of manufactured capital in Daly's model. Other examples of simplification are the explicit or implicit assumption of sustainably managed resources and ecosystems, as well as the negligence of waste and pollution or the assumed constancy of the respective stocks. The model and analytical results presented in Section 6 provide a first reference for the evaluation of other models with more simplifying assumptions about technology, resources and pollution.

Yet, to provide an adequate analytical framework, a comprehensive model of sustainable development must include the above mentioned extensions of the original Solow model, as well as critical limits of basic human needs and ecosystem integrity—where potentially irreversible changes could appear—and the dynamics of population growth and land use change. The suitable inclusion of critical limits and population and land use dynamics may be crucial for analyzing the question about limiting scale from an integrated perspective of both intertemporal efficiency and equity. This indicates an important further direction of research.

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